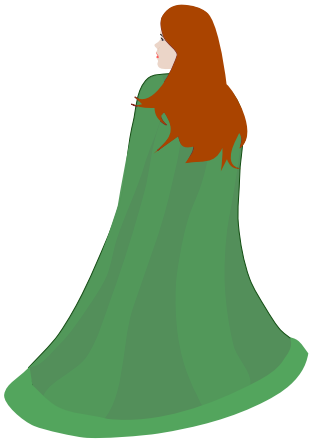
Brigid's new cloak

Brigid has a green cloak. She would like to make a new one using four different colours of fabric.



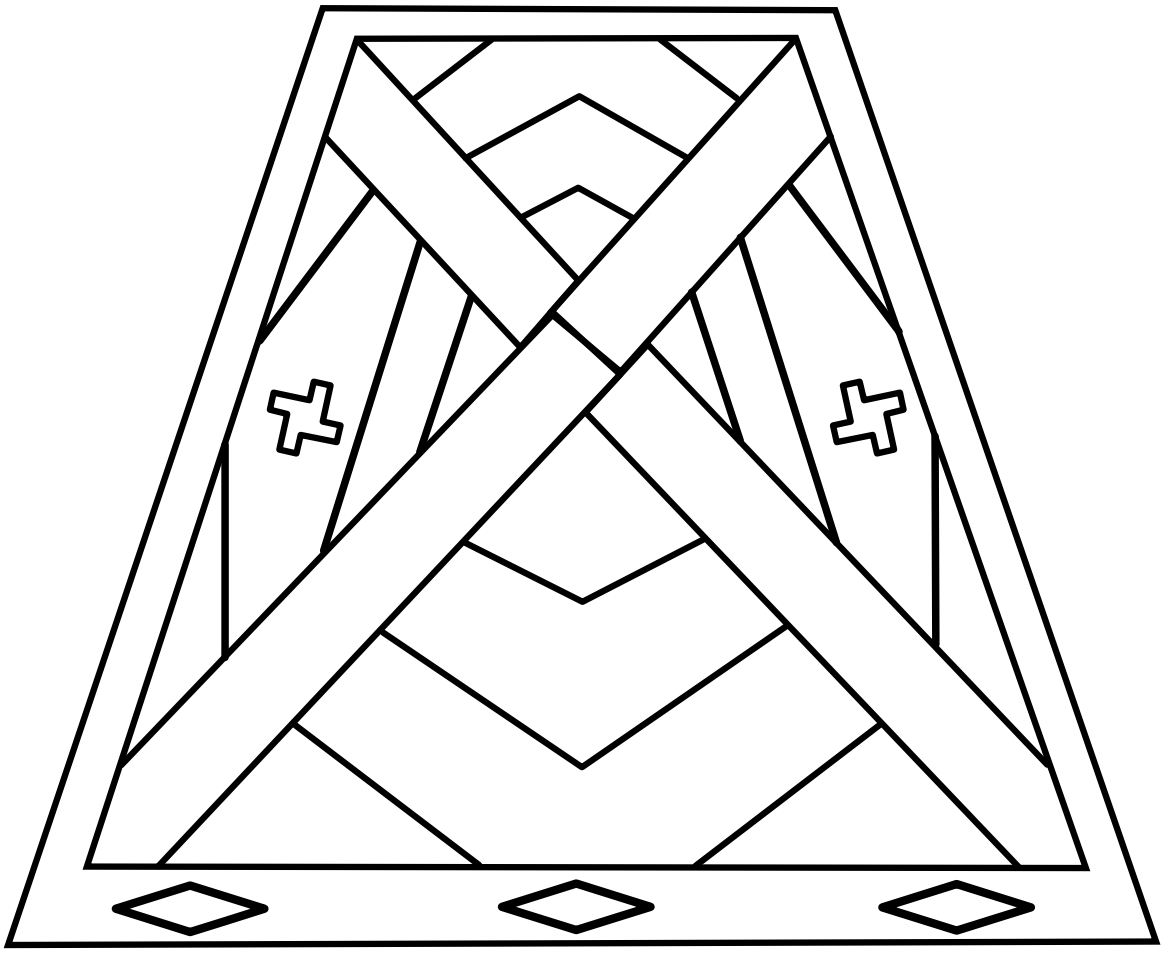
She wants her cloak to be made so that if pieces of fabric touch each other they will have a different colour.

She has made a plan for all of the shapes of the pieces of fabric.

QUESTION

Can you colour in the pattern for Brigid's new cloak using only four colours?

Remember: Where pieces of fabric touch they can not have the same colour.



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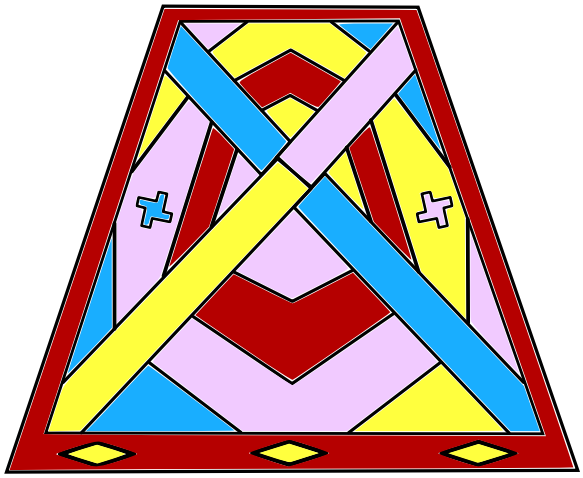
Answer

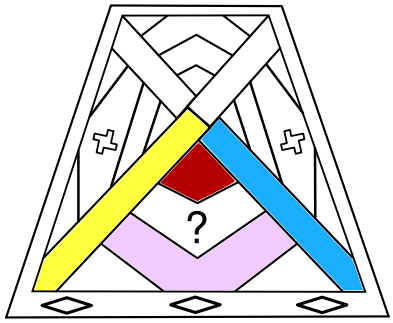
A correct answer is:



There are several other correct answers.

Explanation of the answer

It is possible to get stuck while filling in colours. Filling in four different colours around any piece leaves us stuck without any choices of colour that will work.



We can avoid getting stuck by following these guidelines:

- first choosing colours for the outermost piece and the four crossing pieces that form a large "X" shape at the centre,
- for the remaining pieces that have multiple neighbours, only choosing a piece to colour that has three neighbours already coloured with three different colours.

Incorporating these guidelines, we decide on the following four steps to solve the task.

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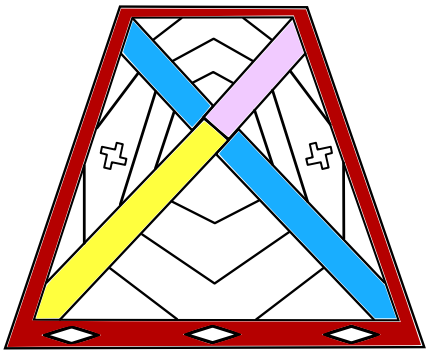
Step (1). Colour in the outermost piece and four crossing pieces of fabric that touch each other at the centre. Any four colours can be chosen. However, because we have five pieces and only four colours, two of the pieces must have the same colour.

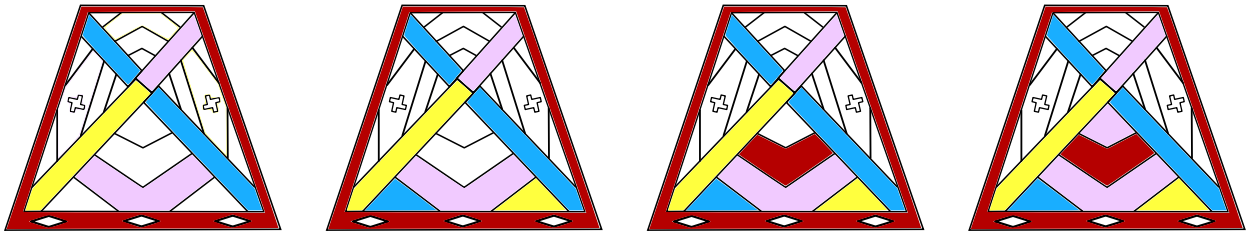
- The outermost piece has to be a different colour from the other four pieces because it touches each of the others.
- The two crossing pieces sloping from lower left to upper right also have to be a different colour from the other four pieces because they also touch each of the others.
- The remaining two pieces can be the same colour because they are the only two of the five pieces that do not touch each other.



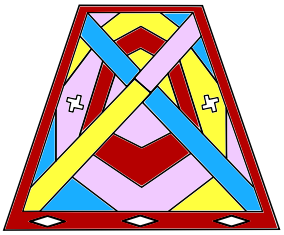
Now to solve the problem of the other pieces of fabric. We make two observations before continuing with the colouring.

i. Since the crossing pieces are now coloured, we notice that the remaining pieces are now divided into four separate “triangle” regions. Since they are separated, we decompose the problem into four separate problems and colour these regions one at a time.
ii. Further, we recognise that except for a tiny detail, the four regions follow the exact same pattern (each has five pieces arranged in an identical way to form a triangle). When we figure out how to colour one triangle, we can generalise the pattern to colour the others.

Step (2). Look at any one of the four remaining “triangle” regions. Find a piece that touches three already coloured pieces, where the three colours are different. Colour that piece (there will be no choice in what colour to choose – we must choose the remaining unused colour). Now, at least one other piece will be touching three coloured pieces. Colour that piece, and repeat for each of the remaining pieces in the “triangle”.



(3) Colour in the other three “triangle” regions using the same pattern (the colours will not be identical, but if you look carefully you will see they form the same pattern relative to the colours of the crossing pieces that enclose them).



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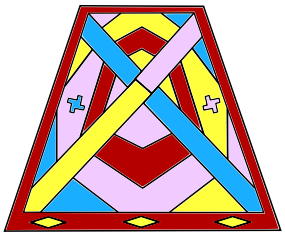
(4) The final small pieces can be any other colour different from the piece they are touching.



There are many different ways to correctly colour this cloak, but all of them follow the same identical pattern. Except for the tiny detail pieces in the cloak, any two pieces that are the same colour in the solution above will the the same in any correct solution to this particular cloak.

Connection to computational thinking

Decomposition: This task is an example of *breaking into parts*. We split the problem into two problems that must be solved in a particular order: first the border and crossing pieces, and secondly all of the other pieces. Then we decompose the latter into four separate "triangle" problems than can be solved in any order. Each time the smaller problems become more manageable. Combining the separate results gives us the colouring for the whole cloak.

Algorithms: This task is an example of *inventing an algorithm*. The requirement to colour the border and crossing pieces first may seem arbitrary, but it is consistent with a heuristic that colours first the pieces that touch the most other pieces (analogous to the jigsaw puzzle heuristic of completing the corners and edges first). After that, solving the problem without ever getting stuck requires one to invent and follow a straightforward algorithm that includes finding and colouring any uncoloured piece that touches three different colours.

Pattern Recognition: This task is an example of *figuring out similar/common elements*. By noticing that the pieces in each of the four regions are touching in the same way, we can use the solution for one region to more quickly find a solution for the other three regions. On a finer scale, recognising which uncoloured pieces touch three different colours requires pattern recognition.

Logic: This task is an example of logical reasoning. Some pieces of fabric are forced to be coloured a particular colour if they have three different colours of fabric already touching them. We used logic when we decided which two of the crossing pieces must be the same colour, and when we coloured the "triangle" regions.