Jigsaw puzzle

Helen wants to assemble a toy that looks like the one pictured on the wall of the toy shop. She needs to make the toy from the eight shapes offered on the counter. Each shape has a different price on it, ranging from 1 coin to 7 coins. She can buy as many shapes as she wants of each type and rotate them in any way. Shapes can touch each other, but not overlap.

QUESTION

What is the minimum cost for Helen to assemble the desired toy?

A. 20 coins

B. 16 coins

C. 13 coins

D. 14 coins

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Answer

The correct answer is C: 13 coins.

Explanation of the answer

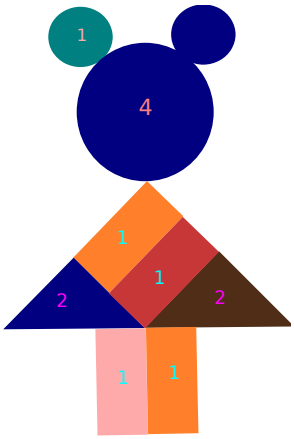
Very often in problems, a good strategy is to break down a complex problem into smaller parts. We start by observing that the shapes that potentially make up the figure of the toy can be one of two fundamental categories: they are either angular or rounded. On the basis of this, the figure can be broken up into two main parts: the head, only containing rounded shapes, and the body (torso and legs), consisting of angular shapes. The body can then be further decomposed into two parts: the torso, which may contain a square, rectangles, triangles or parallelograms, and the legs, which may only consist of rectangles, a square, or a square constructed from triangles. The next step is to determine the minimum number of coins needed for each part.

The head is the easiest part: using the shape identical to the head (6 coins) is more expensive than using a combination of the two other rounded shapes (5 coins in total).

The legs are similarly easy to determine: using two rectangular shapes (2 coins in total) for that part is far cheaper than using one square shape (5 coins). It is also cheaper than using 2 triangles (4 coins).

The most difficult part is the torso because there are many possible combinations of shapes. The best way to proceed is to use the cheapest shape types as much as possible, trying to build the torso from them. We start with the rectangular shape type for the middle of the torso and finish the sides of the torso with two triangles.

Following this decomposition of the problem, we eventually arrive at the least expensive combination in the list of options, which is 13 coins. A separate proof can be used to show that the toy cannot be assembled with fewer than 13 coins.



Connection to computational thinking

Decomposition: This task is an example of *considering different decompositions*. We can split the picture into a head part and a body part. We can then consider the different ways to make up each part. Both parts can be made in many ways, and each needs to be checked.