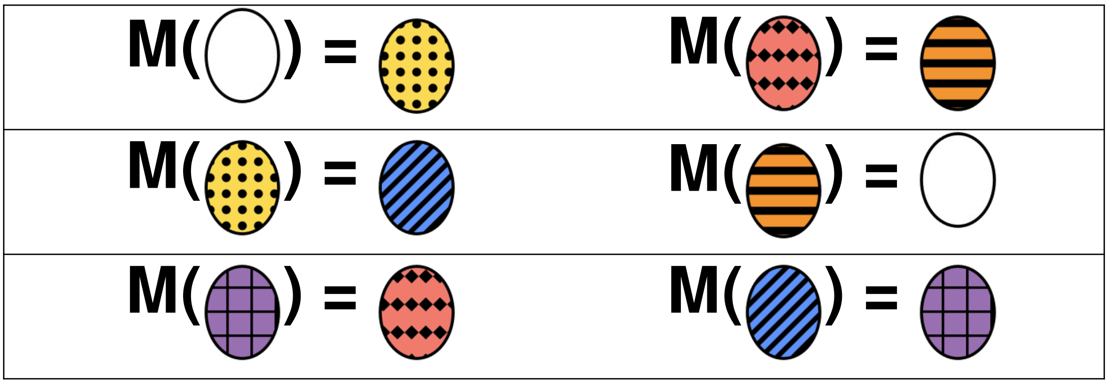
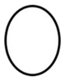
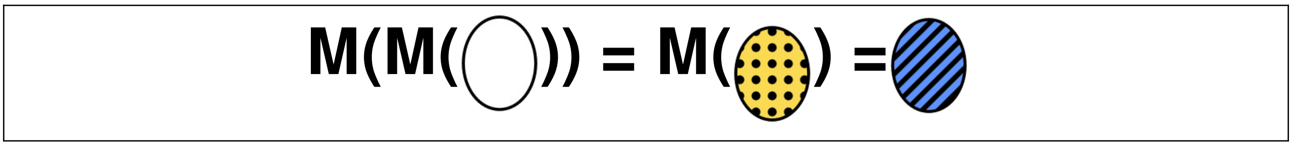
Easter egg painting machine

Ben is a famous engineer who has built a machine called **M** for repainting Easter eggs. If an egg of a particular pattern is put into a machine, it recognises that pattern and repaints the entire egg with a new pattern. Below is a description of how **M** repaints eggs.



Eggs can be repainted several times, for example starting with a plain egg , a first painting gives a dotted yellow egg  , and a second repainting turns the yellow dotted egg into a striped blue egg .This double painting can be presented by this formula:



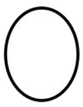
QUESTION

Ben’s loyal fan Elena sent this instruction for painting. Which pattern egg will they get?

A

B

C



D

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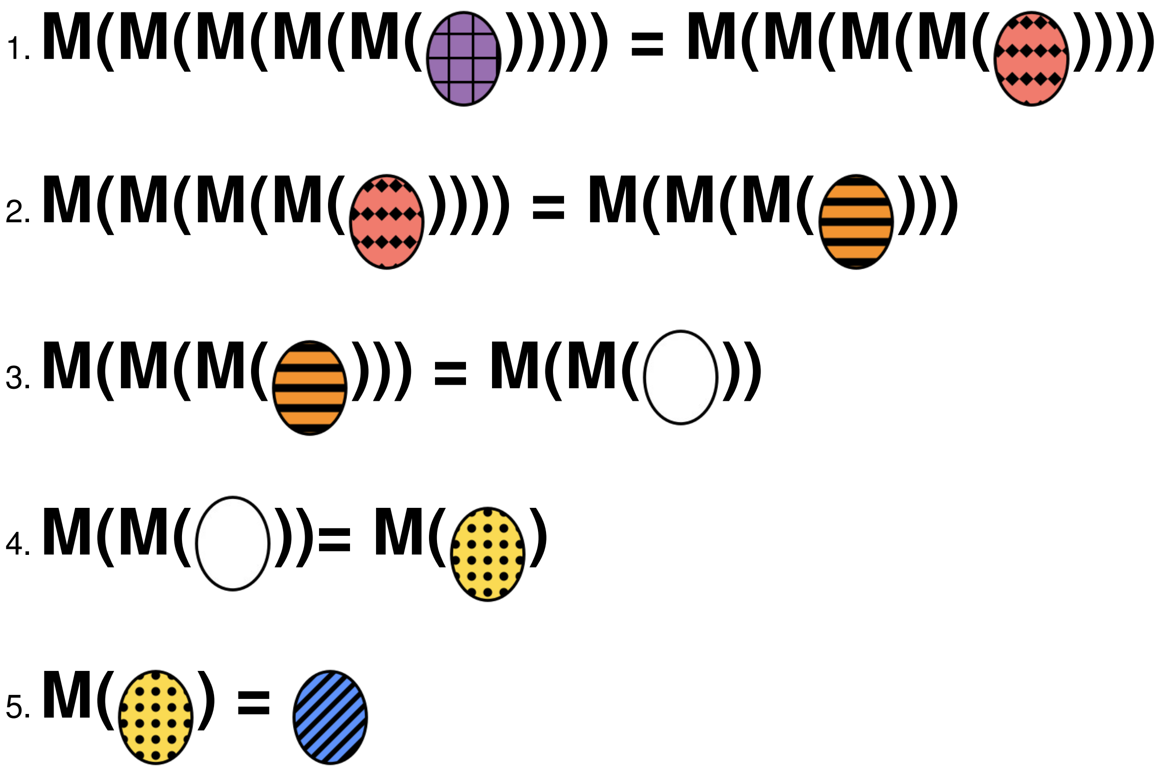
Answer

The correct answer is A.

Explanation of the answer

Based on the task, we can easily conclude that the machine repaints the egg five times. Following the rules, we arrive at the solution step by step.



All other answers offered are not correct.

B) is an incorrect answer. It shows the result of only 4 repaint operations.

C) is an incorrect answer. It shows the result of only 3 repaint operations.

D) is an incorrect answer. It shows the result of only 1 repaint operation.

Connection to computational thinking

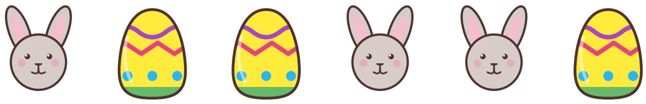
Algorithms: This task illustrates the concept of *following an algorithm*. One is given the inputs and corresponding outputs of a function and are asks to follow recursive calls to that function. Recursion is a programming technique in which a function calls itself in order to solve a problem. The key idea behind recursion is to break a problem down into smaller, more manageable sub problems that are similar to the original problem.

Bunnies and eggs

Sequences of bunnies and eggs are represented by codes, using the following two steps.

1. Moving from left to right in a sequence, each bunny or egg is represented by the number of eggs to the right of it in the sequence. This gives a sequence of numbers.
2. Then each even number in this sequence is replaced with a 0 and each odd number with a 1, to obtain its code.

Let's look at the following sequence as an example.



- The leftmost bunny has 3 eggs to the right of it, so it will be represented with a 3.
- The next egg has 2 eggs to the right of it, so it will be represented with a 2.
- The next egg has 1 egg to the right of it, so it will be represented with a 1.

Continuing this process gives the sequence 3, 2, 1, 1, 1, 0. Its code is 101110.

QUESTION

Which of the following sequences of bunnies and eggs is represented by the code 11101000?

A. 

B. 

C. 

D. 

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Answer

The correct answer is C.

Explanation of the answer

In the codes, a 0 means there is an even number of eggs to the right of that position in the sequence, and a 1 means there is an odd number of eggs to the right. We can go through the given code step by step.

Step 1: Eliminate options using the first digit
The first digit is 1, which means there must be an odd number of eggs to the right of the first position.
The sequences in Options A and C both have 3 eggs to the right of the first position, so their first digits would be 1. The sequences in Options B and D have 4 eggs to the right of the first position, so their first digits would be 0. Therefore, the given code cannot represent Options B or D, and we only need to consider Options A and C.

Step 2: Distinguish between Options A and C
The sequences in Options A and C are the same up to the fifth position.
In Option A, there are 2 eggs to the right of the fifth position, which would give a 0.
In Option C, there is 1 egg to the right of the fifth position, which would give a 1.
The given code has a 1 in the fifth position, so Option A can be eliminated.

Step 3: Verify the correct answer
For the sequence in Option C, counting the eggs to the right gives 3, 3, 3, 2, 1, 0, 0, 0.
Replacing odd numbers with 1 and even numbers with 0 gives the code 11101000, as required.

Connection to computational thinking

Abstraction: In this task, the sequence of bunnies and eggs is simplified to a sequence of numbers, and then to a sequence of 0s and 1s. Instead of working with the full visual arrangement, we focus only on the number of eggs to the right of each position. This shows how information can be reduced to its essential features while still keeping what is needed to recover the original sequence.

Algorithms: The method for creating the code follows a clear set of steps. First, we count the number of eggs to the right of each position. Then we replace each number with 0 or 1, depending on whether it is even or odd. This step-by-step process is an example of an algorithm. Solving the task involves applying this algorithm carefully.

Representation: This task shows how information can be encoded in binary using 0s and 1s. Even though the original sequence is visual, it can be represented in a different way while still conveying the same information. Being able to move between the sequence and its code reflects how computers store and process data.

Easter island egg

Captain Rapa is looking for a buried golden egg on Mini Easter Island. The island is divided into 16 regions, each marked with a letter from *A* through *P* as shown.

Captain Rapa can enter any amount of regions into a special device and the device will tell her whether or not the egg is in one of those regions. For example, if she enters regions *A*, *C*, and *D* into the device and the device shows "no", then the egg is not in regions *A*, *C*, or *D*. When she enters all regions *A* through *P* into the device, the device shows "yes", indicating that the egg is in one of those regions.

QUESTION

What is the minimum number of times Captain Rapa needs to use the device to know for sure which region contains the egg?

A.	B.	C.	D.
3	4	5	6

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Answer

The correct answer is B. 4.

Explanation of the answer

Suppose Captain Rapa enters letters of half of the regions into the device first. Then,

- if the device indicates that the egg is in the given regions, the captain now further divides these regions into two halves and enters one half into the device;
- if the device indicates that the egg is not in the given regions, the captain divides the other regions into two halves and enters one of the halves into the device.

Captain Rapa continues this process until she finds the region that has the egg. Since there are 16 regions to start with; after she uses the device for the first time, there will be 8 regions remaining that might contain the egg. After she uses the device for the second time, there will be 4 regions remaining that might contain the egg. After she uses the device for the third time, there will be 2 regions remaining that might contain the egg. After she uses the device for the fourth time, she will have determined which region contains the egg.

Advanced

The answer explanation would be finished if 4 was the smallest of the multiple answers. However, although we have shown that 4 uses of the device is sufficient to find the egg no matter where it is buried, we need to show that answer A (3 uses of the device) is not correct. We will therefore explain why 4 is the minimum number of times Captain Rapa needs to use the device.

In the following explanation, we call all the regions that might contain the egg, the *remaining regions*. At the start, all 16 regions are the remaining regions. Each time Captain Rapa uses the device, she divides the remaining regions into two smaller groups: those she enters letters for and those she does not enter letters for. One of these smaller groups then becomes the new group of remaining regions.

We will show that using any strategy other than the one described above requires Captain Rapa to use the device more than 4 times.

Now, if Captain Rapa enters a number of letters not equal to half (8) the number of regions into the device on the first go, then the number of remaining regions could be at least 9. For example, if she enters the letters of 7 regions into the device and receives the answer “no”, then there will be 16 – 7 = 9 remaining regions. Or, perhaps she enters the letters of 9 regions and receives the answer “yes” in which case there will then be 9 remaining regions. If she enters any number of regions less than 8 or more than 8 then there could be 9 or more remaining regions.

Since there is no guarantee that the number of remaining regions is less than 9 after using the device once, Captain Rapa could be faced with the possibility that there are 9 or more remaining regions. In that case, when Captain Rapa uses the device a second time, these 9 remaining regions will be divided into two groups and at least one of these groups will have at least 5 regions. Similarly, after Captain Rapa uses the device a third time, she cannot guarantee there will be fewer than 3 remaining regions. And after using the device a fourth time, she cannot guarantee there will be fewer than 2 remaining regions, thus requiing a fifth use of the device.

Since we have shown that every strategy other than giving the device exactly half (at the start, 8) of the remaining regions requires more than 4 uses of the device, and 4 uses of the device is sufficient, we can conclude that 4 uses of the device is the minimum needed.

Connection to computational thinking

Logic: This task illustrates the concept of *logic*. One must appreciate that giving half the regions to the device will reduce the remaining regions by the largest amount in the worst case.

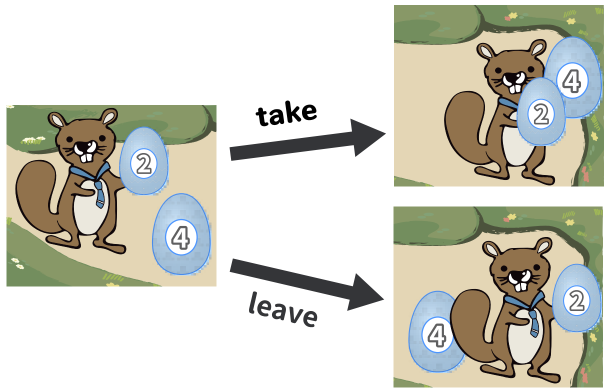
Algorithms: To a lesser extent, this task illustrates the concept of *inventing an algorithm*. If one is not familiar with the algorithm for binary search then one needs to invent an algorithm that repeatedly reduces the number of remaining regions by half at each step.

The great egg-scape

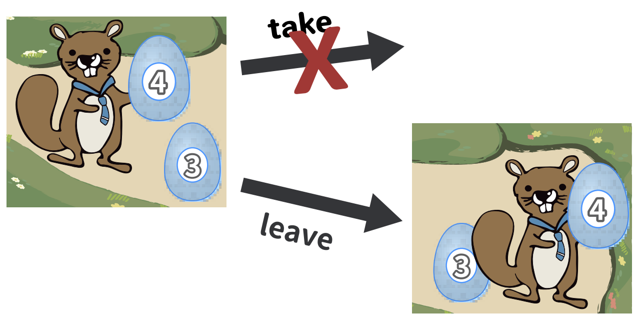
Beaver Deana enters the maze below, carrying an egg of size 1. She wants to go through the maze and collect the eggs that are scattered around.

Deana must move in the direction of the arrows and obey the following two rules each time she finds an egg.

RULE 1: If the egg is bigger than the biggest egg she already carries, she can either take it with her or leave it behind.



RULE 2: Otherwise, she must leave the egg behind.



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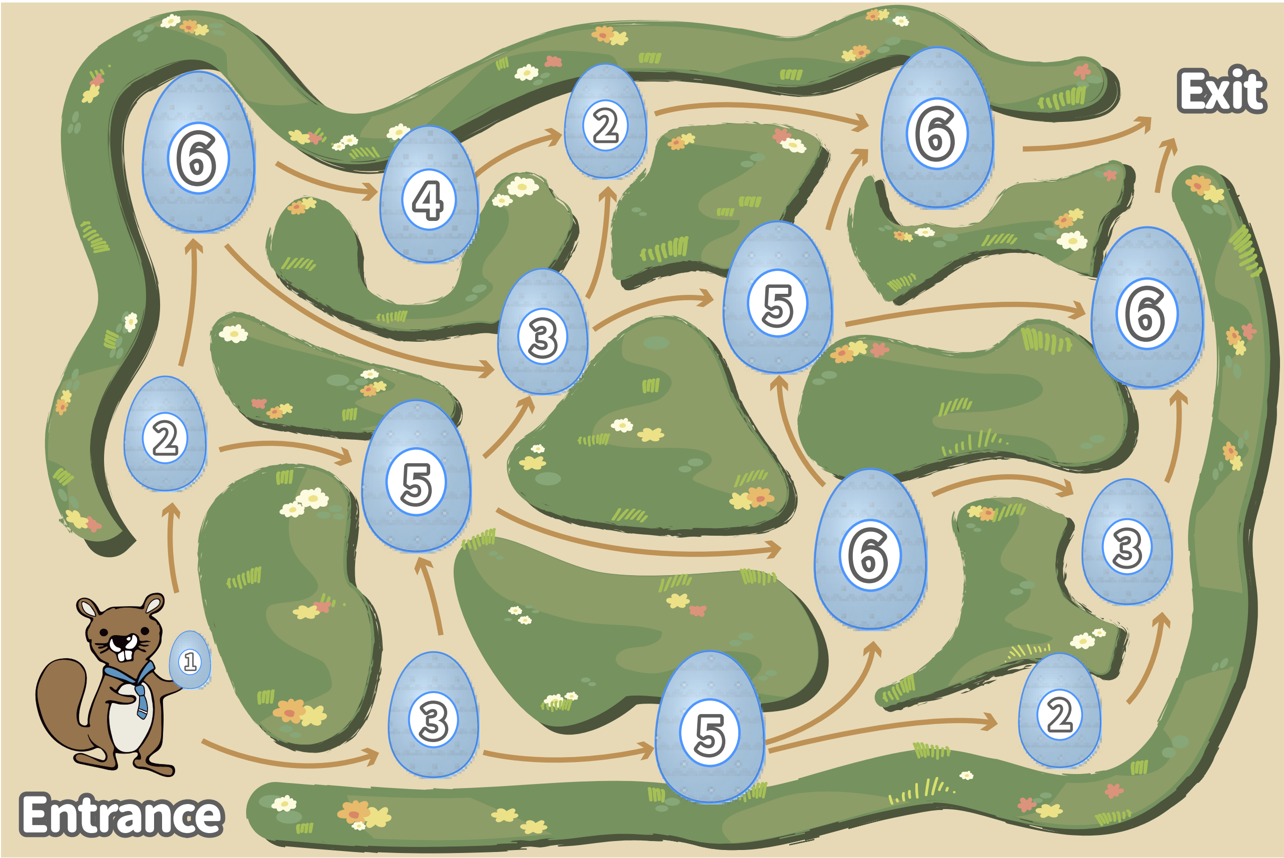


QUESTION

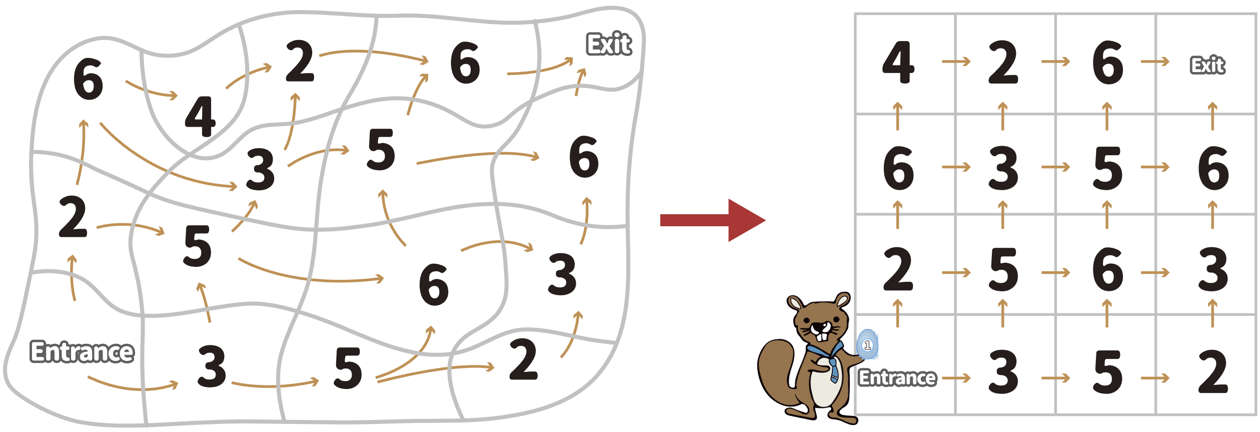
What is the maximum number of eggs that Deana can carry with her, including the size 1 egg, when she gets out the maze?

Answer

The correct answer is 5.

Explanation of the answer

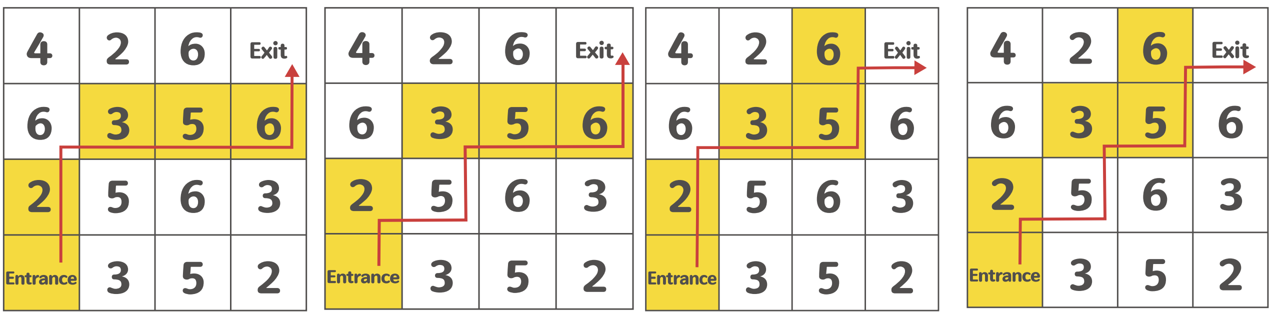
We can draw a simplified map of the maze. Notice that all arrows move either right or up.



All paths from entrance to exit are 5 steps long so the maximum number of eggs she can carry is 6. By the rules, they must be of sizes 1,2,3,4,5,6 and picked up in that sequence.

Notice that it is not possible to pick up a egg of size 5 after picking up the (only) egg of size 4, so Deana cannot carry both 4 and 5 out of the maze. This means the answer is at most 5.

Here are the four possible ways to carry five eggs out of the maze.



Connection to computational thinking

This task asks to find the longest increasing sequence of egg sizes across all paths from the entry to the exit. For a single path, the longest increasing subsequence problem is a standard algorithmic problem with an efficient solution using dynamic programming. Searching for the longest such sequence across multiple paths is a more complex problem. In this maze, the paths have a simple structure (all of equal length, going only up and right) and our simple analysis was sufficient to find the answer.

For more complicated mazes, a more general approach is needed to systematically explore all possible paths. Playing a game like chess or Go involves solving a similar problem. We have to search through all possible sequences of moves to determine which leads to the best outcome. In most cases, it is not feasible to explore all paths, so we try to identify moves that will not be useful to reach a good outcome and stop exploring such paths as we go along.

This task requires **algorithmic thinking**, the ability to efficiently search through a set of possible answers and find the optimum.